MUSIC-Based Multiple CFOs Estimation Methods for CA-OFDM Systems

Chin-Yi Chang∗, Yu-Ting Sun† and Meng-Lin Ku‡
Department of Communications Engineering, National Central University, Taiwan, R.O.C
985203010@cc.ncu.edu.tw∗, 965403001@cc.ncu.edu.tw†, mlku@ce.ncu.edu.tw‡

Abstract—Orthogonal frequency division multiplexing (OFDM) is a very promising technique for high data rate transmission in multipath channels. In general, the achievable data rate of an OFDM system is confined by the available bandwidth, which strongly depends on the spectrum policy. Carrier aggregation is an attractive and flexible bandwidth extension technique which allows for upgrading data rates by aggregating several contiguous or non-contiguous carriers. Different from the conventional OFDM systems, multiple carrier frequency offsets (CFOs) have to be taken into account in carrier-aggregated OFDM (CA-OFDM) systems. In this paper, a set of preambles is constructed as pilots for component carriers. Based on the received preambles, we investigate novel multiple CFOs estimation methods using MUSIC or root-MUSIC algorithms to find multiple CFOs. Two CFOs mapping methods are then proposed to map the estimated CFOs to the corresponding component carriers. Simulation results show the proposed method can get good performance, in terms of the detection error rate and mean square error (MSE) in multipath time-varying channels.

Index Terms—Carrier aggregation, MUSIC algorithms, root-MUSIC algorithms, OFDM, CFO estimation.

I. INTRODUCTION

The explosive growth of multimedia data service has pushed the need for high data rate transmission. To achieve efficient transmission under limited and precious spectrum resource is now becoming more and more important. Orthogonal frequency division multiplexing (OFDM) has received great interest over the past decade since its advantage of bandwidth saving and low complexity implementation in multipath environments. Therefore, it is regarded as a very essential technology for the next-generation wireless communications and chosen as a candidate for a variety of wireless communication standards such as 802.11p, 802.16m, worldwide interoperability for microwave access (WiMAX) and long term evolution/ long term evolution advanced (LTE/LTE-A), etc.

Although OFDM techniques can efficiently improve the spectrum efficiency, it is still very difficult to allot a large bandwidth for providing peak target data rates in excess of 1Gbps under the current spectrum utilization policy. For example, the bandwidth used for the LTE system is only 20MHz, and the maximum available data rate is at most 100Mbps. To overcome this limitation, the concept of bandwidth extension has been proposed in some standardized wireless communication systems by aggregating those unused bands to fit the demand of a larger bandwidth. In the fourth-generation mobile communications, this idea is referred to as spectrum aggregation [1] and carrier aggregation [2] respectively in WiMAX and LTE-A standards. In vehicular Ad hoc networks (VANET), a large amount of information exchange is required for inter-vehicle communications, and to provide an adequate spectrum bandwidth is one challenging issue. Thus, many studies exploit cognitive radio techniques to explore the underutilized spectrum resource; however, the available spectrum bands are usually not continuous [3]-[5]. In this regard, it is necessary to apply the carrier aggregation technique for supporting high data rate communications. These carriers may lie in the same or different bands and may have different bandwidths [6]. Because of the flexibility in using multiple frequency bands, the carrier aggregation technique allows for exploiting numerous carriers at the same time so as to realize high data rate transmission.

Despite the popularity of OFDM techniques, it is far more vulnerable to carrier frequency offset (CFO), as compared with the conventional single-carrier techniques. The mismatch or instability of the local oscillators between the transmitter and the receiver inevitably causes CFOs at the receiver side, which could introduce the intercarrier interference and thus dramatically degrade bit error rate performance. Therefore, it is essential to carefully deal with the frequency synchronization problem, resulted from the mismatch/instability of the oscillators between the transmitter and the receiver, and to compensate the CFOs in the design of OFDM systems. Since the carrier aggregation techniques aggregate multiple carriers for concurrent transmission, the receiver suffers from different CFOs corresponding to different component carriers. To deal with multiple CFOs at the receiver side is one key challenge in this kind of CA-OFDM systems. Particularly, a fast synchronization algorithm for estimating multiple CFOs is necessary for VANET applications since the mobile communication environments could be rapidly changed. In this paper, a physical (PHY) layer multiple CFOs estimation method for CA-OFDM systems is proposed. Conventional CFO estimation methods in OFDM systems can be classified into two categories: cyclic-prefix-based (CP-based) methods [7] and pilot-based methods [8]. These two kinds of methods however can only estimate one CFO, and they cannot be directly applied to estimate multiple CFOs at the same time. A non-continuous carrier aggregation scenario is taken into account in a recent work [9] and a block type pilot-based synchronization method is proposed, whereas the proposed method must use numerous receivers to estimate multiple CFOs.
In this paper, we resort to multiple signal classification (MUSIC) algorithms to estimate multiple CFOs at the same time. Two novel MUSIC-based CFOs estimation methods which employ MUSIC or root-MUSIC algorithms [10] are proposed for estimating multiple CFOs. First, we propose a set of preambles which are constructed from Hadamard sequences, and each component carrier is associated with a particular preamble. A bank of bandpass filters and oscillators is applied to down-convert the received preamble signals into the baseband. Subsequently, we develop two MUSIC-based CFO estimation methods, based on the MUSIC and root-MUSIC algorithms, to estimate the multiple CFOs. Finally, two CFOs mapping methods are proposed to identify which estimated CFOs belong to which component carriers.

The remainder of this paper is organized as follows. In Section II, an CA-OFDM system, along with the preamble structure, is described. The MUSIC-based CFOs estimation methods for carrier aggregation systems are proposed in Section III. Simulation results are discussed in Section IV. Finally, some concluding remarks are given in Section V.

II. CA-OFDM SYSTEMS

A. System Model

Fig. 2 shows the system model for CA-OFDM systems. The system consists of \( N_c \) different component carriers, and each component carrier is associated with a specific frequency band without overlapping with others. Each component carrier may appear as carrier to legacy users in ordinary OFDM systems while users in the carrier aggregation scenario are able to transmit and receive several component carriers simultaneously. The transmitted signals in time domain for the \( m \)th component carrier can be expressed as

\[
x_m(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_m(k)e^{j\frac{2\pi \epsilon_m}{\Delta f} n}
\]

where \( N \) is the number of subcarriers, \( X_m(k) \) is the frequency-domain signal at the \( k \)th subcarrier, \(-N_g \leq n \leq N - 1\) and \( N_g \) is the length of the cyclic prefix (CP). Let \( h_m(l) \) be the complex channel gain of the \( l \)th path corresponding to the \( m \)th component carrier. Further, we denote \( \epsilon_m \) as the normalized CFO for the \( m \)th component carrier. Note that the CFO is mainly due to the oscillator mismatch problem between the transmitter and the receiver, and the normalized CFO is usually defined as \( \epsilon_m = \frac{\Delta f_m}{\Delta f} \), where \( \Delta f_m \) denotes the discrepancy of carrier frequencies for the \( m \)th frequency band, and \( \Delta f \) represents the subcarrier spacing. For practical applications, the maximum CFO could be as large as 20ppm, and the central frequencies of component carriers depend on the frequency bands used for aggregated transmission. For example, in a
typical cellular system operating in 3.5GHz band, the central frequencies of the component carriers usually range between 3.5GHz and 3.6GHz. If the subcarrier spacing $\Delta f$ is 15kHz and the CFO is 20ppm, the maximum normalized CFO is given by $\varepsilon = \frac{20\text{ppm} \times 3.6\text{GHz}}{15\text{kHz}} = 4.8$. In fact, the normalized CFO could be any value ranging between 0 and 4.8.

By assuming the perfect timing estimation and after the CP removal, the received signals in time domain can be expressed as

$$r(n) = \sum_{m=0}^{N_c-1} e^{j2\pi mn} \sum_{l=0}^{L-1} h_m(l)x_m((n-l)) + w(n)$$

where $((\cdot))_N$ denotes the modulo-$N$ operation, $w(n)$ denotes the complex white Gaussian noise with zero mean and variance $\sigma_w^2$, and the channel frequency response corresponding to the $m$th component carrier is given by

$$H_m(k) = \sum_{n=0}^{N-1} h_m(n)e^{-j2\pi kn/N}$$

(3)

B. Preamble Structure

Here, we propose preamble signals which are sent by the aggregated frequency bands to estimate the multiple CFOs at the receiver side. The $P$ Hadamard sequences with the length of $P$ are utilized to construct the preambles in frequency domain, each of which is then mapped onto the frequency band of a specific carrier component for transmission. We assume the total number of component carriers is less than $P$. Denote $S_m(p)$ and $X_m(k)$ as the $m$th Hadamard sequence and the preamble sequence in frequency domain for the $m$th component carriers, respectively, for $p = 0, \ldots, P-1$ and $k = 0, 1, \ldots, N-1$. The $m$th Hadamard sequence is mapped onto the equispaced subcarriers with the distance $Q$ as follows:

$$X_m(k) = \left\{ \begin{array}{ll} S_m(p) & \text{if } k = M + pQ \\ 0 & \text{otherwise} \end{array} \right.$$  \hspace{1cm} (4)

where $M$ is an initial subcarrier index such that the Hadamard sequence is placed within the useful band, and $p = 0, \ldots, P-1$. By choosing a proper value of $Q$ such that $\Omega = \frac{N}{Q}$ is an integer number, from (4), the preamble signals in time domain has the following periodic property:

$$x_m(q + \mu \Omega) = x_m(q)$$

(5)

where $q = 0, 1, \ldots, \Omega - 1$ and $\mu = 0, 1, \ldots, Q - 1$

C. Received Signals

For the convenience of explanation, we ignore the noise term in (2), and define the received preamble signals as

$$\tilde{r}(n) = \sum_{m=0}^{N_c-1} e^{j2\pi mn} \left\{ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_m(k)X_m(k)e^{j2\pi nk} \right\}$$

(6)

Substituting (4) into (6), we can further rewrite received preamble signal as

$$\tilde{r}(n) = \sum_{m=0}^{N_c-1} e^{j2\pi mn} \left\{ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_m(k)X_m(k)e^{j2\pi nk} \right\}$$

$$= \sum_{m=0}^{N_c-1} e^{j2\pi mn} \sum_{k=0}^{N-1} H_m(k)X_m(k)e^{j2\pi nk} + w(n)$$

$$\triangleq \sum_{m=0}^{N_c-1} r_m(n)$$

(7)

Accordingly, the received preamble signals have the following periodic structure:

$$\tilde{r}(q + \mu \Omega) = \sum_{m=0}^{N_c-1} e^{j2\pi mn(q + \mu \Omega)} \left\{ \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} H_m(k)X_m(k)e^{j2\pi nk} \right\}$$

$$= \sum_{m=0}^{N_c-1} e^{j2\pi mn(q + \mu \Omega)} r_m(q)$$

$$\triangleq \sum_{m=0}^{N_c-1} e^{j2\pi mn(q + \mu \Omega)} r_m(q)$$

(8)

where $q = 0, 1, \ldots, \Omega - 1$ and $\mu = 0, 1, \ldots, Q - 1$. Thus, it is noted in (8) that the received preamble is endowed with a periodic structure, i.e., $\tilde{r}(q + \mu \Omega) = \tilde{r}(q + \eta \Omega)$ for any non-negative integer number $\mu$ and $\eta$. Also, from (2) and (7), we can express the received preamble signals including the noise term as

$$r(n) \triangleq \tilde{r}(n) + w(n) \triangleq \sum_{m=0}^{N_c-1} r_m(n) + w(n)$$

(9)

III. MULTIPLE CFOS ESTIMATION

A. MUSIC-Based CFOs Estimation

Based on the above periodic property of the received preamble signals, we now intend to estimate the multiple CFOs by applying MUSIC and root-MUSIC algorithms [10]. The main idea of both MUSIC and root-MUSIC algorithms is to find out the signal subspace and the noise subspace via eigenvalue decomposition of the autocorrelation matrix of the received signals. After these two subspaces are identified, a frequency estimation function is used to find out the component frequencies from the noise subspace. First, we define $r(q) = [r(q), r(q + \Omega), \ldots, r(q + (Q - 1) \Omega)]^T$ and rearrange the received signals of (9) into a matrix form to get the
modified Harmonic model as follows:

\[
F = \begin{bmatrix}
  r^T(0) & r^T(1) \\
  \vdots & \vdots \\
  r^T(\Omega - 2) & r^T(\Omega - 1)
\end{bmatrix}
\]

where \((\cdot)^T\) represents the transpose operation. From (10), the correlation matrix can be estimated and calculated by

\[
\hat{R}_r = \frac{1}{\Omega} F^H F
\]

By applying the eigen decomposition, the signal subspace and the noise subspace can be found in the following:

\[
\hat{R}_r = \frac{1}{\Omega} F^H F = [U_s, U_z] \begin{bmatrix}
  \Lambda_s & O \\
  O & \Lambda_z
\end{bmatrix} \begin{bmatrix}
  U_s^H \\
  U_z^H
\end{bmatrix}
\]

where \(\Lambda_s\) and \(\Lambda_z\) are two diagonal matrices whose diagonal elements are the eigenvalues of \(\hat{R}_r\) arranged in a descending order. That is, we have \(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_q\). Moreover, \(U_s\) and \(U_z\) are the eigenvectors of \(\hat{R}_r\), corresponding to \(\Lambda_s\) and \(\Lambda_z\), respectively. Let \(Q_m(e^{j2\pi f})\) denotes \(v^H(f) u_m\) and define a function \(R_{\text{music}}(e^{j2\pi f})\) with \(f\) as a variable:

\[
R_{\text{music}}(e^{j2\pi f}) = \frac{1}{\Omega} \sum_{m=\Omega+1}^{N_c} \frac{|Q_m(e^{j2\pi f})|^2}{|v^H(f) U_s^H e^{j2\pi f}|^2}
\]

where \(v(f) = [1, e^{j2\pi f}, \ldots, e^{j2\pi(Q-1)f}]^T\) and \(u_m\) is the \(m_{th}\) eigenvector of \(\hat{R}_r\). For the MUSIC algorithm, the multiple CFOs are estimated by finding the \(N_c\) largest values of \(R_{\text{music}}(e^{j2\pi f})\) through one-dimensional search. The other method is to apply the root-MUSIC algorithm for estimating multiple CFOs. In this algorithm, we first find the \(N_c\) roots of a function \(R_{\text{root-music}}(z)\) which are closest to and inside the unit circle, and the function is given by

\[
R_{\text{root-music}}(z) = \sum_{m=\Omega+1}^{N_c} Q_m(z) Q_m^*(\frac{1}{z})
\]

where \(Q_m(z)\) is the \(z\)-transform of \(Q_m(e^{j2\pi f})\) and \(R_{\text{root-music}}(z)\) is the \(z\)-transform of the denominator of the function \(R_{\text{music}}(e^{j2\pi f})\). The multiple CFOs are then estimated by calculating the phases of these \(N_c\) roots. After executing the MUSIC and root-MUSIC algorithms, we can get the \(N_c\) effective CFOs, denoted by \(\theta_m = \frac{\lambda_m}{\Omega} \) for \(\theta_m \in (-0.5, 0.5)\) and \(m = 0, 1, \ldots, N_c - 1\). According to (8), we can know that after MUSIC and root-MUSIC estimation, the estimated phase \(\theta_m\) and the normalized CFOs \(\xi_m = \theta_m \cdot Q\). We also know from (8) that the choice of \(Q\) in (4) is very important as it could affect the affordable estimation range and accuracy of the multiple CFOs. It is noted that no matter how many CFOs need to be estimated, the complexity of two proposed methods is almost identical since the MUSIC and root-MUSIC algorithms are able to estimate multiple CFOs at the same time, whereas the complexity for the conventional CFO estimation method strongly depend on the number of CFOs to be estimated.

**B. Two CFOs Mapping Methods**

After executing MUSIC of root-MUSIC algorithms, the next step is to identify which estimated CFOs belong to which component carriers.

1) **Method I**: Since the preamble signals are constructed from the orthogonal Hadamard sequences, we can identify the estimated CFO belonging to which component carriers via comparing the \(N_c\) cross-correlation functions between the received preamble signals and the transmitted preamble signals with different CFO compensation. Then, the CFO is mapped to a particular component carrier by finding the one with the maximum peak value of the cross-correlation functions. The correlation functions can be calculated as

\[
D(m, l) = \{r_m(n)\} \odot \{e^{-j2\pi f_m l} \cdot x_m^*(n)\}, l = 0, 1, \ldots, N_c - 1
\]

where \(m\) is the component carrier index, \(r_m(n)\) is the received signal for the \(m_{th}\) component carrier, and \((\cdot)^*\) takes the conjugate operation. Finally, the normalized CFO \(\xi_m\) for the \(m_{th}\) carrier component can be inferred from the maximum of \(|D(m, l)|\), for \(l = 0, \ldots, N_c - 1\).

2) **Method II**: Since each estimated CFO can only belong to one component carrier, the Method I has an incidence to map an estimated CFO to two or more component carriers due
to the imperfect cross-correlation property of the Hadamard sequences. We proposed a refined CFOs mapping method, called Method II to improve this drawback, and the detailed procedures are summarized in Fig. 3.

IV. SIMULATION RESULTS

We use computer simulation to demonstrate the performance of the proposed schemes. The length of Hadamard sequences and the FFT size are set to be $P = 32$ and $N = 2048$, respectively. Here, it is assumed that five components are aggregated for data transmission, i.e., $N_c = 5$, and component carriers are associated with different preambles for which the initial subcarrier index $M$ and the subcarrier distance $Q$ are given by 769 and 16, respectively, in our simulation. The typical urban (TU) channel profile specified in COST259 is adopted for simulation. Two performance metrics are included for evaluating the performance of the proposed methods. First, we say the CFOs mapping method have correct detection if all the estimated CFOs can exactly correspond to their own frequency bands. Second, the mean square error (MSE) of the estimated CFOs for all component carriers is presented to verify the synchronization accuracy.

Fig. 4 shows the detection error rate of the two CFOs mapping methods at vehicle speed=120km/hr. It is observed that the Method II outperforms the Method I for both MUSIC and root-MUSIC algorithms. Furthermore, the detection error rate for the MUSIC and the root-MUSIC algorithms are comparable under a selected CFOs mapping method. Since the performance of the MUSIC and the root-MUSIC algorithms is almost identical, we only simulate the MSE performance for the case of the root-MUSIC algorithm with the Method II in the following. Fig. 5 shows the MSE performance of the five component carriers in TU120 channel at vehicle speed 120km/hr. We can observe that the MSE performances of five component carriers are all smaller than $10^{-3}$ when the signal-to-noise ratio (SNR) is larger than 3dB. Fig. 6 shows the MSE performance for the component carrier 3 in TU channel at different vehicle speeds. In this figure, it is shown that our proposed method can provide a good MSE performance even when the vehicle speed is up to 120km/hr.

V. CONCLUSION

In this paper, we focus on the multiple CFOs estimation for CA-OFDM systems. The preambles using Hadamard sequences are constructed in frequency domain for component carriers, and two novel MUSIC-based CFOs estimation methods are investigated to simultaneously estimate multiple CFOs in an efficient way. For the conventional CFO estimation method, the increase in complexity is proportional to the number of component carriers. Our proposed methods however take the advantage of low complexity to estimate multiple CFOs at the same time, and they just slightly increase the complexity when the number of component carriers becomes large. Finally, the performance of the proposed methods has been confirmed through computer simulations, and the results
show that our proposed methods work well in multipath fading channels.

REFERENCES


